The effect of centreline particle concentration in a wave tube

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The interaction of sound waves with an aqueous suspension of solid particles was analysed experimentally and theoretically. A heretofore unreported effect of particle concentration in the vicinity of a wave-tube centreline was observed. The phenomenon is related to the combined effect of Rayleigh-type acoustic streaming, jet-like streaming (quartz wind) and drift forces occurring in the presence of a sonic wave in the suspension-filled tube.

1. Introduction

The interaction of sonic and ultrasonic sound waves with dispersions (suspensions, emulsions, aerosols, etc.) can be used for separating the dispersed particles from the fluid. It is well known that such an interaction produces steady forces on the particles. These forces drive the particles toward the antinode or node planes of the applied standing wave.

An expression for the drift force was first obtained by King (1934) on the basis of an ideal compressible fluid model. This force leads to the so-called radiation drift related to the radiation pressure. The validity of his findings was verified experimentally by Rudnick (1951). A more accurate expression for the radiation drift force was derived by Westervelt (1951) by calculating the difference between the momentum of the incident wave and that of a wave scattered on a sphere. Recently, Fittipaldi (1979) and Higashitani (1981) regarded the drift as an effect which concentrates particles in a standing wave. The drift of aerosol particles in a plane standing wave under the action of Stokes viscous forces was studied by Duhin (1960). Westervelt (1950) considered the drift connected with periodic changes in the viscosity of the medium. Czyz (1987) examined the motion of an aerosol particle in a standing wave field under the influence of drift and resistance forces in the Stokes and Oseen approximation. One should also mention the drift due to distortions of a sinusoidal acoustic wave (Westervelt 1957). Nigmatulin (1990) regarded the effect of the added-mass force upon steady particle drift using multiphase flow equations. An equation of average particle motion, valid for relatively large particles and high frequencies, was derived by means of the small-parameter method. It was shown that the particles on the average can move toward nodes or antinodes of fluid velocity depending on fluid-to-particle density ratio.

Vainshtein, Fichman and Pnueli (1992) have used multiphase flow equations to show that in a vessel with dimensions not exceeding the sonic wavelength particles move away from or toward the oscillating wall depending on the fluid-to-particle density ratio. The equation of average particle motion and corresponding expression for the drift force were derived by means of the small-parameter method. These equations are



FIGURE 1. Circular tube filled with a water suspension of solid particles. Domain with coordinates.

valid for relatively small particles and low frequencies when the friction Stokes force is a dominant interphase force. In a subsequent study Dain *et al.* (1994) have found that a slight curvature of the stationary wall causes the particles to drift sideways. This study was recently extended to high-frequency waves (Dain *et al.* 1995).

Hager & Benes (1991) investigated the effect of a standing wave with a frequency of f = 0.67 MHz in a closed tube of 20 mm in diameter filled with a suspension of small glass spheres (diameter $d \approx 60-160 \,\mu\text{m}$, density $\rho_p = 2.320 \,\text{kg m}^{-3}$) in water. The experiment has demonstrated that particles concentrate at the node planes.

The propagation of ultrasonic sound waves in tubes results in another very interesting physical phenomenon that can affect dispersions, namely acoustic vortical streaming. Kundt (1868) (see Rayleigh 1945) carried out his experiments in a so-called wave tube. Fine sand or lycopodium seed were shaken over the interior of a glass tube containing a vibrating column of air. Dust patterns were formed, by means of which it was possible to determine the positions of the nodes and to measure the distances between them. Kundt's experiments demonstrate that a standing sound wave causes vortical aerosol particle motion periodic with respect to the tube axis. The theoretical study of acoustic streaming started with the work of Rayleigh (1945), who considered the vortex flow which occurs in a long pipe as a result of the presence of a longitudinal standing wave. This work was continued by Westervelt (1953), Nyborg (1953), and Schlichting (1955), Lighthill (1978) has stressed the fundamental role of acoustic energy dissipation in the evolution of the gradients in the momentum flux, which bring about the secondary streaming. He analysed many types of acoustic streaming including jetlike winds generated by powerful ultrasonic sources. Stuart (1966) has introduced the streaming Reynolds number, R_s , based on the characteristic velocity of the secondary flow. In contrast to the previous investigations which considered $R_s \leq 1$, he studied cases where $R_s \ge 1$. An analysis of the acoustic streaming field around spheres was started by Riley (1966).

The secondary acoustic streaming in a narrow cell caused by a vibrating wall has been investigated by Vainshtein, Fichman & Pnueli (1995). The case when the wavelength is many times the gap of the cell has been analysed. Rayleigh's streaming at large R_s has been treated by Vainshtein (1995). These papers also contain reviews of recent works on the acoustic streaming phenomenon. One should also note a recent experimental investigation of streaming flow around acoustically levitated samples by Trinh & Robey (1994).

In the present paper we investigate the effect of sonic waves (f = 20 kHz) on a suspension of polystyrene particles ($\rho_p = 1003 \text{ kg m}^{-3}$, $d \approx 50-100 \text{ µm}$) in water. The experiments are carried out in a glass tube with an inside radius of 25 mm. The results demonstrate a centreline-concentration effect when the particles gather in the vicinity of the points of intersection of the tube axis and the node planes, and move as clusters

from one stable point to another. A theoretical investigation based on the comparative analysis of the effects of Rayleigh-type acoustic streaming, jet-like streaming and the drift forces is undertaken to explain this result. The conditions of the experiments of Kundt (see Rayleigh 1945) and Hager & Benes (1991) in which such an effect was not detected are analysed from the point of view of the approach developed.

2. Analysis

Consider a monodisperse rarefied suspension of spherical particles of diameter d inside a circular tube of length L, and radius R (figure 1). A plane cosinusoidal standing sound wave is imposed in the longitudinal x-direction:

$$U = u_0 \cos nx \cos \omega t, \quad n = \omega/c, \tag{1}$$

where U(x, t) is the external-flow velocity, u_0 the characteristic amplitude of oscillations, ω a typical frequency, c the speed of sound, and t time. We assume that the frequency is below the 'cut-off' frequency for radial oscillations in a tube with resilient walls

$$\omega < 2.41 \ c/R. \tag{2}$$

The presence of the standing wave inside the tube leads to the occurrence of two physical phenomena connected with the interaction of a sound wave with the boundaries of the tube and the solid particles occupying its interior. These phenomena are vortical streaming and steady particle drift. We consider the combined effect of these phenomena on particle motion in the tube. Initially, each of these phenomena will be treated separately.

2.1. Acoustic streaming

Acoustic streaming was first studied by Rayleigh (1945). It occurs in the second approximation with respect to the wave amplitude; its characteristic feature is that the sonic wave adjacent to a solid wall is dissipated within the resulting Stokes periodic boundary layer. The effective thickness of the layer is given by

$$l = (2\nu/\omega)^{1/2},$$
 (3)

where ν is the kinematic viscosity. This thickness is assumed to be much smaller than the sound wavelength,

$$\lambda = 2\pi c/\omega,\tag{4}$$

i.e.

$$\lambda \gg l. \tag{5}$$

In view of the last condition, and since the velocity in this layer, as in the sound wave itself, is much less than that of sound, the flow may be regarded as incompressible.

The equation of the periodic boundary layer may be solved by successive approximations with respect to the small parameter u_0/c . The treatment yields in the second approximation a slip velocity u_s , defined as the tangential velocity of the streaming motion just outside the boundary layer. This leads to the law of streaming, resulting in the slip velocity (Schlichting 1955)

$$u_s = u_{s0} \sin 2nx, \quad u_{s0} = 3u_0^2 / 8c. \tag{6}$$

The characteristic velocity of acoustic streaming, u_{s0} , may be expressed in terms of the sound energy density, \overline{E} ,

 $u_{s0} = 3\bar{E}/(4\rho_f c),$

$$\bar{E} = \frac{1}{2}\rho_f u_0^2,\tag{7}$$

(8)

as

where ρ_f is the unperturbed fluid density.

The slip velocity, given by (6), serves as a boundary condition for determining the main acoustic flow. In such calculations, the Stokes boundary layer thickness (3) can



FIGURE 2. Rayleigh's acoustic streaming generated in a pipe by a cosinusoidal wave. The total streaming flow is divided into four equal parts by the stream tubes shown. The small solid circle represents the area of the particle point-concentration effect.

be regarded as negligible, and expression (6) taken effectively as a slip velocity at the solid surface. However, the character of the streaming motion so generated outside the boundary layer can still depend on the value of the dynamic viscosity, μ , through the value of the streaming Reynolds number, R_s , introduced by Stuart (1966):

$$R_s = u_{s0} D/\nu, \tag{9}$$

where D is a linear dimension of the system. For streaming with

$$R_s \ll 1 \tag{10}$$

the inertial terms in the Navier-Stokes equations are negligible. Rayleigh's calculation of the velocity profile for a standing wave in a tube of radius R with the slip velocity u_s as a boundary condition yields

$$u = -u_{s0} \left(1 - \frac{2r^2}{R^2} \right) \sin 2nx, \quad v = u_{s0} nR \left(\frac{r}{R} - \frac{r^3}{R^3} \right) \cos 2nx. \tag{11}$$

The x-component of the velocity changes sign at a distance $r = R/\sqrt{2}$.

Rayleigh's acoustic streaming described by these expressions consists of a series of vortices located symmetrically about the axis of symmetry and periodic in the x-direction, with a period $\lambda/2$ as shown in figure 2. The direction of the flow inside the vortices is indicated in the figure by arrows. The normal velocity component, v, is directed toward the walls at the velocity antinode, x = 0, and toward the axis of symmetry at the velocity node, $x = \lambda/4$. The relationship between tube radius and half-wavelength can be arbitrary (Rayleigh 1945; Lighthill 1978).

We now define the various characteristic times connected with the process at hand:

$$t_s = \frac{1}{nu_{s0}} = \frac{8c^2}{3u_0^2\omega}, \quad \tau = \frac{\rho_p d^2}{18\mu},$$
 (12)

where t_s is the characteristic time of acoustic streaming, τ is the Stokes relaxation time of particle motion, ρ_p is the intrinsic density of the particle material.

Let the Stokes number be introduced:

$$St = \tau/t_s. \tag{13}$$

We consider the case when St is small

$$St \ll 1.$$
 (14)

In this case the particles in the tube follow the fluid in the zeroth approximation and also accomplish the same vortical motion.

The vortices of a fine-sand aerosol in a tube were observed by Kundt (see Rayleigh 1945). His experimental results were explained by Rayleigh (1945) whose theoretical analysis, with some modifications, were presented above.

2.2. Steady particle drift

Sound-induced oscillations in dilute fluid-solid dispersions produce forces exerted by the fluid on the particles. In the case of the inertial particles encountered in the present study these forces include (Nigmatulin 1990): the Stokes drag force, the force connected with an external pressure gradient and the induced mass force. During the interaction of sound waves with particles the hydrodynamic forces produce steady particle motion which is characterized by an averaged force. This is the so-called drift force. Depending on particle size, frequency of oscillations and fluid-to-particle density ratio, the effect of these hydrodynamic forces on the magnitude of the drift may differ. In general one may describe this oscillating drift force, which is the local gradient of the radiation pressure, by the following expression:

$$F_D = F_0 \sin 2n\xi,\tag{15}$$

corresponding to standing wave (1). Here ξ is the particle position averaged over time. The value and the sign of the constant F_0 depend on the mechanism of the occurrence of the particle drift. In the approximation which is valid for small Reynolds numbers based on particle size, the equation of motion of the average position of the particle is (Nigmatulin 1990; Czyz 1987)

$$\frac{d^2\xi}{dt^2} + \frac{1}{\tau}\frac{d\xi}{dt} - A_D \sin 2n\xi = 0, \quad A_D = \frac{F_0}{m_p},$$
(16)

where m_p is the particle mass and A_D denotes the maximum value of the acceleration induced by the drift force. For sufficiently small particles their inertia is negligible, and (16) simplifies to

$$\frac{\mathrm{d}\xi}{\mathrm{d}t} = A_D \tau \sin 2n\xi,\tag{17}$$

which was first introduced in the drift theory by King (1934) and St. Clair (1949). The drift force guides the particles to the closest planes of velocity nodes or antinodes, depending on the sign of the constant F_0 . These planes correspond to stable stationary points of an equation describing the particle drifting motion.

For $F_0 > 0$ particles drift toward nodes:

$$x = \frac{1}{4}\lambda + \frac{1}{2}\lambda k, \quad k = 0, \pm 1, \dots;$$
(18)

for $F_0 < 0$ particles drift toward antinodes:

$$x = \frac{1}{2}\lambda k, \quad k = 0, \pm 1, \dots$$
 (19)

Based on (16), (17), a characteristic velocity of particle drift may be estimated by

$$U_D = |A_D|\tau. \tag{20}$$

Here we consider relatively large frequencies and particles when the following criterion is satisfied (Nigmatulin 1990):

$$\frac{\rho_f}{\rho_p} \omega \tau \gg 1. \tag{21}$$

In this case the interphase friction force is negligible, and the drift connected with the pressure gradient and induced mass force dominates. A description of the asymptotic treatment of the problem with respect to the amplitude of the oscillations is given in Nigmatulin (1990). The multiphase flow model is used when both volume and mass fractions of the dispersed phase are small. First, the fluid flow field in the second approximation, which reflects the oscillating effect, is determined. Then the motion of

rigid dispersed particles is analysed, the equation of the particle drift in the form (16) is derived, and an expression for the drift force is obtained which may be presented in the form

$$A_D = \frac{3E\omega}{2\rho_f c} \left(2 - \frac{\rho_f}{\rho_p}\right). \tag{22}$$

The non-zero contribution to the drift force is provided by the interaction between the fluid-phase acceleration, which is due to the pressure gradient, and the variation of the particle induced mass due to the fluid-phase density alteration. The sign of the drift force depends on the fluid-to-particle density ratio. The force vanishes when $\rho_f = 2\rho_f$. Using (20), (22) and the condition $\rho_f \approx \rho_p$, which is fulfilled in our experiments, one obtains for the characteristic particle drift velocity

$$u_D = 3\omega\tau \bar{E}/(2\rho_f c). \tag{23}$$

In accordance with (22), (23), one has $A_D > 0$; therefore the stable stationary points of (16), are determined by (18), i.e. they correspond to the velocity nodes. Hence, by condition (21), the drift force gathers particles at the plane of the nodes of a standing wave.

2.3. Combined effect

Now, we examine the combined effect of Rayleigh-type acoustic streaming and drift force on particle motion in a standing wave field. When a drift force F_D determined by (16), (22) is applied to the particles, they drift toward the nodes. Let the characteristic acoustic streaming velocity, u_{s0} , be smaller than that of the drift velocity, u_D , but comparable to it:

$$u_D > u_{s0}. \tag{24}$$

In this case the effect of acoustic streaming causes simultaneous particle motion toward the axis of symmetry, so that the particles are gathered at the points of intersection of the node planes and symmetry axis. One of these points (namely $x = \lambda/4$, r = 0) located within one vortical cell is noted in figure 2 by a small black circle. Using (8) and (23), inequality (24) may be rewritten in the form

$$\Pi_u = 2\omega\tau, \quad \Pi_u = u_D/u_{s0},\tag{25}$$

where Π_u denotes the ratio of the characteristic drift velocity to that of acoustic streaming. This expression may be employed for arbitrary values of R_s .

If the characteristic streaming velocity is much less than the drift velocity, i.e.

$$\Pi_u \gg 1, \tag{26}$$

then the influence of acoustic streaming may be neglected. Then the particles will be concentrated at the node planes where their high concentration will reduce the action of acoustic streaming and prevent particle point concentration at intersection points of node planes with the axis of a tube.

In the case $\Pi_u < 1$, the effect of acoustic streaming dominates, and the particles do not concentrate near the node planes, attaining vortex flow.

3. Experimental results and discussion

We now describe an experimental investigation of standing sound wave propagation in a tube filled with a suspension of polystyrene in water, with the following parameters:

$$L = 1 \text{ m}, \quad R = 0.025 \text{ m}, \quad f = 20 \text{ kHz}, \quad E = 2-5 \text{ J} \text{ m}^{-3}, \\ \rho_f = 10^3 \text{ kg m}^{-3}, \quad c = 1450 \text{ m} \text{ s}^{-1}, \quad \mu = 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}, \\ \rho_p = 1.03 \times 10^3 \text{ kg m}^{-3}, \quad d = 0.5 \times 10^{-4} \text{ m}. \end{cases}$$
(27)



FIGURE 4. Intensity of the sonic field in a tube.

Simple calculations demonstrate that at these values of the parameters the conditions of (2), (5), (14) are satisfied.

The experiments were carried out in degased water, in a Perspex tube of dimensions given above. The experimental set-up is shown in figure 3. The tube was held by two bearings allowing it to rotate and thus preventing particle deposition in the horizontal configuration. The experiments were performed with the tube held either horizontally or vertically, and similar results were obtained for both positions. The suspended polystyrene particles had a concentration of not more than 1 g l⁻¹. A 20 kHz ultrasonic wave was generated by a 600 W, 3.8 cm in diameter piezzo-electric transducer. Opposite the transducer a wave reflector was installed, the position of which could be adjusted in order to change the length of the working space. A pressure sensor for measuring the intensity of the sonic wave could be introduced into the fluid at different points, thus allowing the position of nodes and antinodes to be determined. The wavelength used in the experiments was 7.4 cm. The experiments were carried out for two extreme cases of low and high sonic field intensities. The change of the wave mean intensity with distance from the sonic source is presented in figure 4.

3.1. Low-intensity sonic field

It was observed that in a low-intensity sonic field ($\bar{E} \approx 2 \text{ J m}^{-3}$) particles cluster in swarms near a number of periodic points on the tube centreline. The distance between the points was about half a wavelength. Figure 5 is a photograph of a typical



FIGURE 5. Swarms of particles in a low-intensity sonic field.

experiment, showing swarms of particles concentrating in the vicinity of several points on the tube centreline. We call this effect the *point-concentration effect*.

The point-concentration effect can be explained by the combined effect of acoustic drift and vortical streaming. At the experimental conditions given in (27), one obtains that $\Pi_u = 34.8$. Apparently, in this case a small but persistent effect of acoustic streaming leads to particle concentration at the intersection points of the tube axis and the node planes.

The following question arises: why has the point-concentration effect not been observed in previous experimental works? To answer this question we analyse the conditions of other relevant experimental investigations by means of the approach developed above.

Hager and Benes (1991) investigated the effect of a standing wave on small glass spheres inside a closed water-filled tube. The conditions of the experiments are described in the introduction. In the experiments a uniform particle concentration at the node planes was observed, and the point-concentration effect was not detected. It should be noted that in their experiments frequencies significantly higher than ours were used. For such conditions, at $d = 60 \,\mu\text{m}$ the calculations give $\Pi_u = 1682$, i.e. inequality (26) holds. Hence, in this case the effect of acoustic streaming is negligible.

Now, let us consider the conditions of Kundt's classical experiments (see Rayleigh 1945) on vortical acoustical aerosol motion in tubes. We first analyse the combined effect of acoustic streaming and drift forces on particle motion in the case of fine aerosols. Drift of very small particles in a standing sonic wave of sufficiently low

frequency has been considered by Vainshtein *et al.* (1992) and Dain *et al.* (1994, 1995). The main forces exerted by the fluid on the particles in this case are the Stokes force and the force connected with the pressure gradient. The induced mass force is negligible. In these works the equation of motion of incompressible particles was treated asymptotically with a small parameter $\omega \tau$, the equation of average particle motion in the form (17) was derived and the expression for the drift force obtained in the form

$$A_{D} = \frac{u_{0}^{2}}{4c}\omega\left(2\frac{\rho_{f}}{\rho_{p}} - 1\right) = \frac{\bar{E}\omega}{2\rho_{f}c}\left(2\frac{\rho_{f}}{\rho_{p}} - 1\right).$$
(28)

The drift force vanishes from $\rho_f = \rho_p/2$. For aerosols when $\rho_f/\rho_p \ll 1$ this result coincides with that of Czyz (1990), where A_D is negative. This means that, according to drift theory, aerosol particles have to drift toward velocity antinode planes. Using (20) and $\rho_f/\rho_p \ll 1$ one obtains for the characteristic velocity of aerosol particle drift

$$u_D = \frac{E}{2\rho_f} \omega \tau. \tag{29}$$

Comparing (8) and (29) we find that for sand particles with $d \approx 1 \,\mu\text{m}$ the characteristic velocity of acoustic streaming is larger than that of particle drift, $u_{s0} > u_D$, up to relatively high frequencies. For moderate frequencies the inequality

$$\Pi_u < 1 \tag{30}$$

holds. Under these conditions, fulfilled in Kundt's experiments, the particles do not gather at the antinode planes but move in a vortex motion following acoustic streaming.

3.2. High-intensity sonic field

It should be noted that the steady-state phenomenon shown in figure 5 has been observed only at low acoustic energies. Increasing the energy leads to strong fluid streaming along the centreline in the direction opposite to the oscillating surface. The particles still concentrate near the periodic points, but they move as a swarm periodically from one stable point to another. For these higher intensities ($\bar{E} \approx 5 \text{ J m}^{-3}$) the swarm of particles grows in size. At some critical mass, the swarm spontaneously starts to move along the centreline to the next node point with a lower field intensity. There it coalesces with the swarm which has been there before. After some time this new swarm leaps again to another node point. The process starts at the node nearest to the sonic source. This is the node of highest intensity. The direction of the motion is away from the sonic source along the x-axis. The velocity of the swarm is roughly several centimetres per second. Figure 6 shows several stages in which a swarm moves toward another swarm at a neighbouring node point. The motion of the swarm is induced by the gradient of the acoustic momentum flux which is related to the decrease of the sonic field intensity $I = \overline{E}/c$, as shown in figure 4. We call this effect the centreline-concentration effect. The motion of the swarm can be changed by adjusting the intensity of the sonic field. The swarm can be stopped and even made to move in the reverse direction toward its previous position.

This type of acoustic streaming is analysed by Lighthill (1978). The falling-off in momentum flux creates acoustic streaming, the momentum of which increases as the acoustic momentum flux decreases. One can think of the gradient of acoustic momentum flux as a mean force acting on the fluid. Acoustic streaming is generated by the force, and the particles are just carried along with the fluid. The drag due to this



FIGURE 6. Coalescence of two swarms in a high-intensity sonic field.

detaches the first swarm, and moves it downstream to coalesce with the second. The latter will experience a higher drag, and itself be detached, and so on. If the acoustic beam were exactly parallel then the force acting on the fluid in each cross-section of the tube would be uniform. Automatically, then, a gradient of mean pressure which perfectly opposes that force would be set up. In that case no streaming would be observed. It may be concluded that it is the non-uniformity in the acoustic beam (greater intensity in the centre than at the periphery) which generates the additional acoustic streaming observed. Such a sonic wind became familiar in the thirties when powerful sources based on the piezoelectric properties of quartz began to be commonly used; hence it was sometimes known as the 'quartz wind' (Lighthill 1978).

4. Conclusions

The effect of a standing wave on dispersions of particles was investigated. In the experimental part of the work, the interaction of a standing wave (f = 20 kHz) with a suspension of polystyrene particles ($\rho_p = 1003$ kg m⁻³, d = 50-100 µm) in a water-filled tube (R = 25 mm) was analysed. A new effect of particle centreline concentration was observed. The effect results in particles concentrating in the vicinity of the wave tube centreline.

In a low-intensity sonic field the particles concentrate in the vicinity of the points of intersection of the tube axis and the node planes. To explain this effect, a theory based upon the comparative analysis of Rayleigh-type acoustic streaming and the drift forces was developed. It introduces the parameter Π_u , which represents the ratio of the characteristic drift velocity to that of acoustic streaming. From the qualitative analysis it follows that if $\Pi_u > 1$ the point-concentration effect takes place. If $\Pi_u \ge 1$, the effect of acoustic streaming is negligible, and the particles concentrate near the node planes. If $\Pi_u < 1$ the effect of acoustic streaming prevails, and the particles move in a circular vortex motion.

In the present experiments it was shown that a small but persistent effect of acoustic streaming is quite sufficient for the particles to concentrate at the centreline. The conditions of the classical Kundt experiments correspond to the case $\Pi_u < 1$, and the experiments of Hager & Benes (1991), relate to $\Pi_u \ge 1$. Therefore, in these experiments the point-concentration effect has not been observed.

In a high-intensity sonic field the particles still concentrate near the periodic points, but they move as swarms along the centreline. The jet-like wind sometimes known as the quartz wind (Lighthill 1978) is generated by the ultrasonic source. This strong acoustic streaming carries the particles along with the fluid.

The interaction of standing ultrasonic waves with dispersions of particles can be used for particle separation in water purification processes. Acoustical processing may be indispensable in the case of chemically active or explosive dispersions. The centrelineconcentration effect may be applied in the development of new technologies related to purification of dispersions.

REFERENCES

CZYZ, H. 1987 The aerosol particle drift in a standing wave field. Arch. Acoust. 12, 3-4, 199-214.

- Czyz, H. 1990 On the concentration of aerosol particles by means of drift forces in a standing wave field. *Acustica* **70**, 23–28.
- DAIN, Y., FICHMAN, M., GUTFINGER, C., PNUELI, D. & VAINSHTEIN, P. 1995 Dynamics of aerosol particles in two-dimensional high frequency sonic field. J. Aerosol Sci. 2, 575–594.
- DAIN, Y., VAINSHTEIN, P., FICHMAN, M. & GUTFINGER, C. 1994 Side drift of aerosolse in a twodimensional slowly oscillating sonic field. Aerosol Sci. Technol. 21, 149–156.

- DUHIN, S. S. 1960 Theory of the aerosol particle drift in a standing sonic wave. Colloid J. 22, 128–130. (In Russian).
- FITTIPALDI, F. 1979 Partial coagulation by means of ultrasonics. Acoustica 41, 263-266.
- HAGER, F. & BENES, E. 1991 A summary of all forces acting on spherical particles in a sound field. Proc. Ultrasonic Intl 91 Conf. and Exhibition, Le Touquet, France, 1-4 July, pp. 1-4.
- HIGASHITANI, T. 1981 Migration of suspended particles in plane stationary ultrasonic field. Chem. Engng Sci. 36, 1877–1882.
- KING, L. B. 1934 On the acoustic radiation pressure on spheres. *Proc. R. Soc. Lond.* A 147, 212–240. LIGHTHILL, J. 1978 Acoustic streaming. *J. Sound Vib.* 61, 391–418.
- NIGMATULIN, R. I. 1990 Dynamics of Multiphase Media. Annals of Nuclear Energy, Hemisphere.

NYBORG, W. 1953 Acoustic streaming due to attenuated plane wave. J. Acoust. Soc. Am. 25, 68–75. RAYLEIGH, LORD 1945 Theory of Sound, Vols. 1 and 2, 2nd Edn. Dover.

- RILEY, N. 1966 On a sphere oscillating in a viscous fluid. Q. J. Appl. Maths 19, 461.
- RUDNICK, J. 1951 Measurements of the acoustic radiation pressure on a sphere in standing wave field. J. Acoust. Soc. Am. 23, 633-634.
- SCHLICHTING, H. 1955 Boundary Layer Theory. Pergamon.

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- ST. CLAIR, H. W. 1949 Agglomeration of smoke, fog or dust particles by sonic waves, *Indust. Engng Chem.* 41, 2434–2438.
- STUART, J. T. 1966 Double boundary layers in oscillating viscous flow. J. Fluid Mech. 24, 673-687.
- TRINH, E. H. & ROBEY, J. L. 1994 Experimental study of streaming flows associated with ultrasonic levitators. *Phys. Fluids* 6, 3567–3579.
- VAINSHTEIN, P. 1995 Rayleigh streaming at large Reynolds number and its effect on shear flow. J. Fluid Mech. 285, 249-264.
- VAINSHTEIN, P., FICHMAN, M. & PNUELI, D. 1992 On the drift of aerosol particles in sonic fields. J. Aerosol Sci. 23, 631-637.
- VAINSHTEIN, P., FICHMAN, M. & PNUELI, D. 1995 Secondary streaming in a narrow cell caused by vibrating wall. J. Sound Vib. 180, 529–537.
- WESTERVELT, P. J. 1950 The mean pressure and velocity in a plane acoustic wave in gas. J. Acoust. Soc. Am. 22, 319-327.
- WESTERVELT, P. J. 1951 The theory of steady forces caused by sound waves. J. Acoust. Soc. Am. 23, 312–315.
- WESTERVELT, P. J. 1953 The theory of steady rotational flow generated by sound field. J. Acoust. Soc. Am. 25, 60–67.
- WESTERVELT, P. J. 1957 Acoustic radiation pressure. J. Acoust. Soc. Am. 23, 26-29.